

Tensor Part-VI

Symmetric tensor:

(i) Let us consider a contravariant tensor X^{ij} of rank 2 and if

$$X^{ij} = X^{ji}$$

then $X^{ij} \rightarrow$ symmetric tensor

Similarly for X_{ij} , $X_{ij} = X_{ji} \rightarrow$ symmetric.

(ii) Consider an arbitrary rank tensor, for example

$$X^{ijk}_{lmn}$$

$$X^{ijk}_{lmn} = X^{jik}_{lmn}$$

then X^{ijk}_{lmn} is symmetric in the first two

contravariant indices.

Similarly if $X^{ijk}_{lmn} = X^{ikj}_{lmn}$, then

$X^{ijk}_{lmn} \rightarrow$ symmetric in the first and

third covariant indices.

Antisymmetric tensor:

(i) Consider the previous example, i.e., x^{ij} → tensor quantity and if

$$x^{ij} = -x^{ji} \quad \text{or}$$

⇒ x^{ij} is antisymmetric

Similarly for x_{ij} , if

$$x_{ij} = -x_{ji}$$

then x_{ij} → antisymmetric.

(ii) For tensor x^{ijk}
 l_{mn} if

$$x^{ijk} = -x^{jki}$$

x^{ijk}
 l_{mn} → antisymmetric in the first two contravariant indices.

Again if $x^{ijk} = -x^{ikj}$, then

x^{ijk}
 l_{mn} → antisymmetric in the first and

third covariant indices.

Example of symmetric tensor: —

(i) Kronecker delta → δ_{ij} or δ^{ij} etc

(ii) For an isotropic system, stress tensor σ_{ij} or σ^{ij} is symmetric tensor.